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Kleinian Groups and the geometry of the non-euclidean euclidean algorithm

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Abstract

There is an algorithm for determining when a non-elementary two-generator subgroup of $PSL(2,\mathbb{R})$ is or not discrete. When it is discrete, it is a Fuchsian group and the quotient of the unit disc by the group is a Riemann surface. There are several different ways in which to interpret the algorithm: as a geometric algorithm, as a type of BSS machine or as a symbolic computation algorithm where the entries in the two matrices are algebraic numbers lying in a finite extension of the rationals. The different forms of the algorithm were found by Gilman and/or Jiang to be of polynomial time complexity. In this talk we re-interpret the algorithm as a geometric algorithm in the hyperbolic plane using the non-Euclidean distance. The algorithm then becomes a type of " Euclidean algorithm" using hyperbolic distance, that is, a non-Euclidean Euclidean algorithm. This formulation of the algorithm simplifies the proof of polynomial time complexity. The non-Euclidean Euclidean algorithm has implications for the length spectrum of curves on the surfaces, defining equations of the surface and higher genus surfaces. In this talk we also touch upon implications of the non-Euclidean Euclidean algorithm for algebraic and geometric discreteness criteria in $PSL(2,\mathbb{C})$, that is, Kleinian groups.

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