V Iberoamerican Congress on Geometry
December 2010, Pucón, Chile

Isometric imbeddings of hyperbolic spaces and Riemann surfaces

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Abstract

As early as 1932, Ludwig Bieberbach presents a model of a non singular and complete surface in a separable Hilbert space, which is isometric to the hyperbolic disk; he does so by expanding the hyperbolic metric in a power series. He also establishes the property that the whole group of Moebius isometries of the unit disk corresponds to restrictions of affine isometries of the ambient Hilbert space.

Later Eugenio Calabi in 1953 generalizes this idea to obtain isometric imbeddings of complex manifolds into infinite dimensional spaces. He also points out that analogous mappings exist to map the disk into projective Hilbert space with the Fubini-Study metric, making the observation that affine isometries are the projection of unitary linear mappings.

In this lecture we first show that it is worthwhile to consider specific Hilbert spaces, namely

- $A^2(\mathbb{D})$ the Bergman space of square integrable holomorphic functions in the unit disk, for Bieberbach's imbedding.
- $H^2(\mathbb{D})$ the Hardy space of holomorphic functions in the disk, square integrable in the boundary, for Calabi's imbeddings in projective space.

The formulas obtained there allow us to obtain similar results for imbeddings of

- The unit ball in $\mathbb{C}^n$ with the Bergman metric.
- Siegel's generalized unit disk.
- Hyperbolic three-space.

Furthermore we find an explicit local imbedding of any curve in $\mathbb{P}^2$ into what Calabi calls a generalized Minkowsky space.

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