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Calabi-Yau Siegel threefolds

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Abstract

We describe some Siegel modular threefolds which admit a Calabi-Yau model. Basic for our method is a result of van Geemen and Nygaard. They prove that the complete intersection $\mathcal{X}$ in $P^7(\mathbb{C})$, defined by the equations

$$Y_0^2 = X_0^2 + X_1^2 + X_2^2 + X_3^2$$

$$Y_1^2 = X_0^2 - X_1^2 + X_2^2 - X_3^2,$$

$$Y_2^2 = X_0^2 + X_1^2 - X_2^2 - X_3^2$$

$$Y_3^2 = X_0^2 - X_1^2 - X_2^2 + X_3^2$$

is biholomorphic equivalent to the Satake compactification of $\mathcal{H}/\Gamma'$ for a certain subgroup $\Gamma' \subset \mathrm{Sp}(2, \mathbb{Z})$. This variety has 96 singularities which correspond to certain zero-dimensional cusps and these singularities are ordinary double points (nodes). Cynk and Mayer pointed out that a (projective) small resolution of this variety is a rigid Calabi-Yau manifold $\tilde{\mathcal{X}}$.

We shall consider suitable quotients of this variety that admits a Calabi-Yau model in the following weak sense:

There exists a desingularization in the category of complex spaces of the Satake compactification which admits a holomorphic three-form without zeros and whose first Betti number vanishes.

There are some quotients of this variety that admits a projective Calabi-Yau model.

We will report on these examples.

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