V IBEROAMERICAN CONGRESS ON GEOMETRY
December 2010, Pucón, Chile

Galois closures and Lagrangian varieties

LIDIA STOPPINO

Abstract

Let X be a smooth complex algebraic variety of dimension n and consider the cup product homomorphism $\wedge^2 H^1(X,\mathbb{C}) \longrightarrow H^2(X,\mathbb{C})$ and its holomorphic part $\psi_2\colon \wedge^2 H^0(X,\Omega^1_X) \longrightarrow H^0(X,\Omega^2_X)$. The understanding of these homomorphisms gives several information on the topology of X. For instance, if $\ker\psi_2$ is non-trivial, the fundamental group of X is non-abelian. A classical result due to Castelnuovo-de Franchis states that $\ker\psi_2$ contains decomposable elements if and only if X admits a fibration over a curve of genus greater or equal to 2. The most important case where non-decomposable elements appear in $\ker\psi_2$ is when X is Lagrangian, i.e. it admits a finite map in an abelian variety A of dimension $2n$, such that there exists a 2-form ω on A whose pullback vanishes on X.

In this talk I will describe a joint work with F. Bastianelli and P. Pirola, where Galois closures of finite morphisms, we produce varieties with non-trivial elements in $\ker\psi_2$ - and more generally in any $\ker\psi_k$. Using this method, we construct a new family of Lagrangian surfaces of general type. Our surfaces are the Galois closure of a degree 3 morphism from an abelian surface A with a $(1,2)$ polarization to the Hirzebruch surface $\mathbb{F}_3$. The Albanese variety of these surfaces is the abelian fourfold $A\times A$, and the Lagrangian structure arises from the Albanese morphism. We prove that these surfaces are non-fibred over curves of genus greater or equal to 2. Moreover, we compute the invariants of these surfaces and prove that their topological index is negative, thus disproving a conjecture of Barja-Naranjo-Pirola.