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The random surfaces of a projective curve.

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Abstract

For any fixed projective curve $A \subset \mathbb{P}^{d-2}$ defined over a field $\mathbb{K} = \overline{\mathbb{K}}$, we associate families of "random" nonsingular projective surfaces of general type with Chern numbers asymptotically proportional to certain "log" numbers defined by A. These numbers mainly depend on the position of A in $\mathbb{P}^{d-2}$ with respect to a fixed hyperplane arrangement $\mathcal{H}_d$.

When $\mathbb{K} = \mathbb{C}$, these surfaces have positive index and their fundamental group is isomorphic to $\pi_1(\bar{A})$, where $\bar{A}$ is the normalization of A. Hence, any rational projective curve produces simply connected smooth projective surfaces of general type with positive index. In addition, their Chern ratio is asymptotically close to the "log" ratio defined by A. We want to have this Chern ratio as close as possible to the Bogomolov-Miyaoka-Yau bound 3. We only know precise bounds when A is a line.

In this talk I will explain the two intermediate steps to construct these surfaces. First, given A we produce a ruled surface over $\bar{A}$ with a collection of "divisible" sections. This is clearly organized using Kapranov's construction of the moduli space $\overline{M}_{0,d+1}$. The "log" numbers defined by A are the log Chern numbers defined by its ruled surface together with an arrangement of sections and some singular fibers. These log numbers have constrains directly related to geometric height inequalities. The second step is a sequence of random root covers branch along the arrangement of curves above. The asymptotic result follows from a certain behavior of Dedekind sums and Hirzebruch-Jung continued fractions, now applied to more general singularities.

Along the way, I will show some differences between positive and zero characteristics.