UNIVERSAL LENGTH BOUNDS FOR NON-SIMPLE CLOSED GEODESICS.

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Abstract

We investigate the relationship, in various contexts, between a closed geodesic with self-intersection number k (for brevity, called a k-geodesic) and its length. We show that for a fixed compact hyperbolic surface, the short k-geodesics grow like the square root of k. On the other hand, if the fixed hyperbolic surface has a cusp and is not the punctured disc, then the short k-geodesics grow logarithmically.

The length of a k-geodesic on any hyperbolic surface is known to be bounded from below by a constant that goes to infinity with k. In this paper, we show that the optimal constants $\{M_k\}$ grow like $\log k$. Moreover, we show that for each natural number k, there exists a hyperbolic surface where the constant M_k is realized as the length of a k-geodesic. This was previously known for $k = 1$, where the figure eight on the thrice punctured sphere is the shortest non-simple closed geodesic.