

Combinatorial length, geometric length and self-intersection
of curves on surfaces.

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Abstract

Consider an orientable surface S with boundary and a free homotopy class C of closed oriented curves in that surface. The combinatorial length of C is the minimum number of letters required for a description of C in terms of a set of standard generators of the fundamental group of S . The self-intersection of C is the minimum number of times in which a representative of C crosses itself. If the surface is endowed with a hyperbolic metric, then one can also define the geometry length of C , as the length of the unique geodesic representative in C . I'll discuss several relations between combinatorial length, geometric length and self-intersection number. For instance:

1. what is the statistical distribution of the self-intersection number if a free homotopy class chosen at random from among all classes of a given combinatorial length;
2. what is the statistical distribution of the geometric length if a free homotopy class chosen at random from among all classes of a given combinatorial length;
3. what is the possible maximal self-intersection number for a free homotopy class of given combinatorial length.

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