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Moduli of real and quaternionic vector bundles over a curve.

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Abstract

We examine a moduli problem for real and quaternionic vector bundles over a smooth complex projective curve, and we give a gauge-theoretic construction of moduli spaces for such bundles. These spaces are irreducible subsets of real points inside a complex projective variety. We relate our point of view to previous work by Biswas, Huisman and Hurtubise, and we use this to study the $\mathrm{Gal}(\mathbb{C}/\mathbb{R})$-action $[\mathcal{E}] \mapsto [\overline{\sigma^*\mathcal{E}}]$ on moduli varieties of semistable holomorphic bundles over a complex curve with given real structure σ. We show in particular a Harnack-type theorem, bounding the number of connected components of the fixed-point set of that action by $2^g + 1$, where g is the genus of the curve. In fact, taking into account all the topological invariants of a real algebraic curve, we give an exact count of the number of connected components. We show, moreover, that these connected components are pairwise homeomorphic, thus generalising to rank $r \geq 2$ the results of Gross and Harris on the Picard scheme of a real algebraic curve.